Subject: Electromagnetic Field
Branch: Electrical Engg (III Sem)

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(1) when the vector differential operator ∇ operates vectorally on a vector function, the result is a vector quantity known as curl of vector

if $\vec{A}$ is vector then $\vec{\nabla} \times \vec{A}$ denotes the curl of $\vec{A}$

in Cartesian coordinate

$$\text{Curl of } \vec{A} = \vec{\nabla} \times \vec{A} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$$

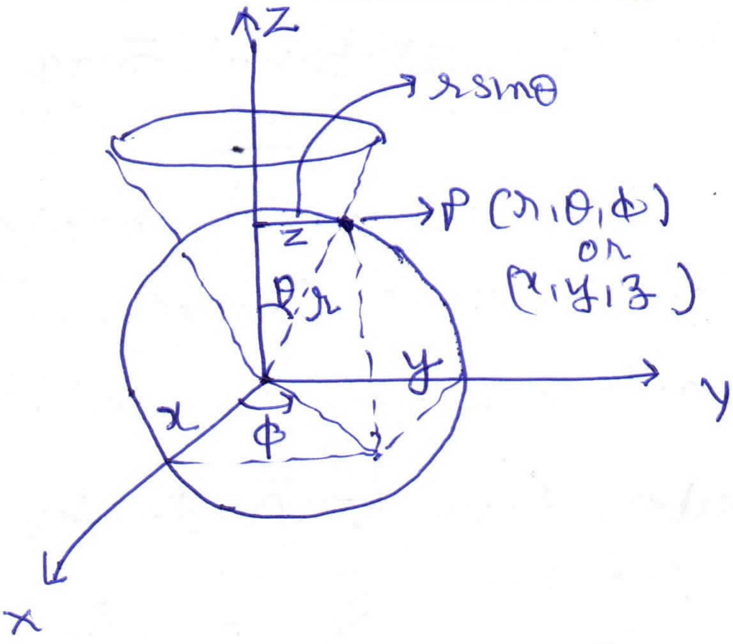
in cylindrical coordinate

$$\text{Curl of } \vec{A} = \vec{\nabla} \times \vec{A} = \frac{1}{\rho} \begin{bmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{bmatrix}$$

and in spherical coordinates

$$\vec{\nabla} \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{bmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{bmatrix}$$

2) Relation b/w Cartesian and spherical co-ordinate!



It is clear from above diagram

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

from spherical to Cartesian system

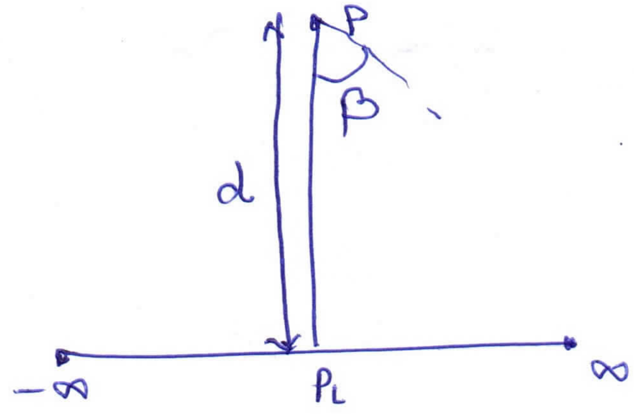
and from these Equation

$$r = \sqrt{x^2+y^2+z^2}, \quad \theta = \cos^{-1}\left(\frac{z}{r}\right)$$

and $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

from Cartesian to spherical system

3) Electric field intensity due to a charge uniformly distributed over an infinite line:



The electric field at Point P

$$|\bar{E}| = \frac{\rho_L}{2\pi\epsilon_0 d} \times \frac{a}{\sqrt{a^2+d^2}}$$

$$|\vec{E}| = \frac{\rho_L}{2\pi\epsilon_0 d} \times \frac{1}{\sqrt{1+\frac{d^2}{a^2}}}$$

if the length of line is form $-\infty$ to ∞ then $a = \infty$

$$\therefore \quad |\vec{E}| = \frac{\rho_L}{2\pi\epsilon_0 d} = \frac{\rho_L}{2\pi\epsilon_0 R}$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\hat{a}_R} \quad \text{or} \quad \boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho}$$

(4) Coulomb's low state that force between two charged bodies is proportional to product of charges and inversely proportional to square of distance b/w them

$$Q_1 \xleftrightarrow{\quad R \quad} Q_2$$

$$F \propto \frac{Q_1 Q_2}{R^2} \Rightarrow F = \frac{K Q_1 Q_2}{R^2}$$

$$\therefore \quad \boxed{F = \frac{1}{4\pi\epsilon} \cdot \frac{Q_1 Q_2}{R^2}}$$

where $\epsilon = \epsilon_0 \epsilon_r$

$\epsilon_0 \to 8.85 \times 10^{-12}$ F/m

$$\boxed{\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon R^2}\hat{a}_R}$$